Fixed Income

Outline of our class on duration, convexity, and DV01.

Overview

Duration is an old-fasioned, nevertheless ubiquitous, way of characterizing the risk of a fixed income security or portfolio. Duration is a measure of price sensitivity to a change in “interest rates.” However, the interest rate is just the security’s yield. And we should really be interested in knowing why and how the security’s yield changes. In fact, I refer to a bond’s price and yield as two sides of one coin. Duration looks at an instantaneous effect, so there is no time adjustment.

Here is the outline of my lecture on duration.

1. Initially focus on 0-coupon securities.
2. There are 2 ways to motivate duration.
   (a) Duration is the negative of the percentage change in price in response to a 100 bp increase in the security’s yield. We measure the percentage of a price in finance as the change in logs. If the price increases from 100 to 110 we say the $ln(110) - ln(100) = 9.531\%$. Note that this measure of percent change is symmetric (Since $ln(110) - ln(100) = ln(110/100) = -ln(100) - ln(110)$. By contrast “arithmetic measures” suffer from an index number problem: $\frac{110-100}{100} = .1$ whereas $\frac{100-110}{110} = -9.09\%$. Since the price of a 0-coupon is e^{-rT}, $ln(P) = -r \cdot T$. So when the yield to maturity changes from r to $r+d$, $ln(P)$ changes to $-(r+d)T$. Note that $dr = r+d-r = d$. And we have the the proportional change in price is $-T * (r + d - r)$, so $\frac{dln(P)}{dr} = \frac{-T}{dr}$.
   (b) Since the price of a 0-coupon is e^{-rT} $ln(P) = -rT$. If r changes to r' then $ln(P') = -r'T$. So we have $\frac{dP}{P} = -Tdr$ and then $\frac{dP}{dr}/P = -T$. Duration is defined as $-\frac{dP}{dr}/P = T$. When we analytically solve $\frac{dP}{dr}$ using the calculus, given $P = e^{-rT}$, we use the chain rule: $\frac{dP}{dr} = -T \cdot e^{-rT}$ (since $\frac{d[e^x]}{dx} = e^x$). Dividing this by P we get $-T$, so again we have $D = -\frac{dP}{dr}/P = T$.
3. We can (and usually should) verify duration numerically.
4. Numerically we can see the effect of convexity - a second-order effect. (And the difference between $\frac{dP}{dr}/P$ and $ln(P') - ln(P)$.) –P is nonlinear function of r.
5. The first thing an anlyst looks at when assessing a fixed income portfolio is its duration. Most professional portfolio managers are evaluated relative to a benchmark. The most common benchmark is the old “Lehman Agg,” now known as the Bloomberg Barclays Aggregate Fixed Income Index. As of October 15, 2020, this index reports a portfolio duration of 6.17. Most fund managers will not deviate from their benchmark’s duration, as they don’t want to bet on the direction of interest rates.
6. One interpretation - to first order approximation, the risk of a portfolio (no matter how complex) with duration 6.17 is the same as a 6.17-year 0-coupon bond.
7. DV01 - One problem of duration is that it has P in its denominator – what about a swap (or any 0-net investment position)? $DV01 = \frac{dP}{dr}/10,000$. So it is -Duration * Price / 10,000. Since $\frac{dP}{dr}$ of a 0-coupon bond is $-T \cdot e^{-rT}$. So the DV01 of a zero-coupon bond is $\frac{-T \cdot P}{10,000}$.
8. We can and should compute DV01 numerically.
9. Portfolio duration and DV01. A FI portfolio can be defined by the weights on each asset in the portfolio. Example, Consider that the yield curve is flat at 4% (cc). We have \$1 million par in a 1-year bill and \$1 million par in a 20-year STRIPS.
   - What is our portfolio value?
     - P(1-year)= 960,789.44
     - P(20-year) = 449,328.96
     - Portfolio value = 1,410,118.40
   - What are the weights on the individual securities?
     - $w_1 = \frac{960,789.44}{1,410,118.40} = 68.14\%$

- $w_2 = \frac{449{,}328.96}{449{,}328.96} = 31.86\%$
- –Because this is a standard portfolio (i.e., not 0-net investment) the weights sum to 1. Consider a swap on its origination date. The weights are 1 in the receiver side and -1 for the pay side. Here the weights sum to 0.

- Now what is the duration / DV01 of our portfolio? It is $w_1 \cdot D_1 + w_2 \cdot D_2$ (since the portfolio value is linear in the weights, the analytics simply transfers the wts to the derivatives. $D_1 = 1$ and $D_2 = 20$, so we have $w_1 \cdot D_1 + w_2 \cdot D_2 = 7.05$. So to a first-order approximation, this portfolio has the same risk as a 0-coupon with a 7.05 year term.

  Compare the 2 numerically. Our portfolio: at flat yield curve of 5A 7.05 0-coupon security with the same initial value, the initial price (at 4the par value is 1,869,504.97. When the yc rises to 5%, the value of this position becomes: 1,314,128.46, for a log change of -7.05%. Why does the "barbell portfolio" do better? A: Convexity.

10. Convexity: Proportional second derivative of P wrt r: T^2. So the convexity of the bullet portfolio is 49.7025 and the barbell is 128.14. More convexity: less negative reaction to rate increase and more positive reaction to rate decrease.

    (Repeat analysis for a drop in ytm from 4 to 3%.)

    For the same reasons portfolio convexity is the sum of the component weights * each component's convexity.

    A portfolio can have positive or negative convexity. Examples of negative convexity: 0-net investment long bullet short barbell mortgage backed paper (Why?)

11. Coupon bond duration.
    Important intuition: Coupon bond is a portfolio of 0-coupon bonds.
    Duration follows portfolio logic.

12. Swap DV01.

13. Common use of DV01 is to hedge. (A 0 DV01 position).

14. Unnecessary complexities:

    - Macualay Duration: The same idea as our duration, but the yield is semi-annually compounded, so the value is very close, and it's a wee bit more complicated algebraically. (Scale by $\frac{1}{1+\frac{r}{2}}$). Our approach is what quant shops use.
    - Convexity means little as a number – it is often scaled. Best to know whether you're long convexity – in which case you're likely to be paying for some insurance.
    - We follow Tuckman (the "Bible"), who uses DV01 and Duration to refer to yield-based measures – i.e., we computed them numerically by adding 100 bps or 1 bp to the security's ytm., or analytically by looking at the derivative with respect to the security's yield to maturity.

      PVBP (or sometimes, PV01) – means add 1 bp to all "fitting securities' " yields – that would be all the 0-coupon rates used to value a coupon bond, for example.
    - Note that under this definition, the PVBP of a 0-coupon note equals its DV01. These terms are often used loosely, but we follow the Tuckman "Bible." (Many references use PVBP and DV01 interchangeably.)