Fixed Income
Problem set on dynamic duration hedging of an annuity – 2 –

Short Answer questions. Be concise, precise and direct.

1. Write a VBA code that generates 10,000 draws from a $\mathcal{N}(100, 20)$ distribution and write out the sample mean and sample standard deviation of these draws. Use the inverse cumulative distribution function for a $\mathcal{N}(0, 1)$ to get the necessary Z variates.

2. *Quiz Implications: You should be able to write and explain the VBA code required to do the preceding exercise. Specifically you should be able to explain the inverse cumulative distribution method used to obtain random draws from a N(0,1) distribution. The best answer would be to write down and explain the graph that shows the projection of the uniform(0,1) draw onto the cumulative distribution function to get Z. You should know the properties of Z and how to convert Z into a $\mathcal{N}(\mu, \sigma)$ variate.*

3. Write a VBA code that simulates the interest rate path and provides a plot of the yield over time from the following process: (Hint: to plot the path, write the time period and the rate to the spreadsheet (in 2 columns) and then have Excel produce an XY scatter plot of the simulated path.) Start the process at 4% (i.e., $y_0 = .04$) and plot it for 20 years.

$$y_t = y_{t-1} + \sigma \cdot \sqrt{\Delta t} \cdot Z_t; \quad Z_t \sim \mathcal{N}(0, 1)$$
$$y_t \geq .005$$
$$y_t \leq .015$$

   For the following cases:

   (a) $\sigma = .025$ and $\Delta t = .25$ (i.e., 3 months).

   (b) $\sigma = .03$ and $\Delta t = .5$ (i.e., 6 months).

   In all cases compute the mean and standard deviation of the interest rates and the changes in interest rates. Outside of the plotting, do Monte Carlo to assess the Monte Carlo mean and standard deviation of these 4 statistics–using 10,000 draws of the time series of rates.

4. Modify the above processes by tightening and widening the cap and the floor of the process, and evaluate the effects of these changes on the Monte Carlo mean and standard deviations of the interest rate levels and changes in the interest rate.

5. *Quiz Implications: The equation above is a model that describes the dynamics of the yield curve. It is not consistent with the absence of arbitrage in financial markets because it requires the yield curve to always be a horizontal line. You should be able to explain how this model works – what is the source of randomness, what is the effect of changing σ, Δt, the cap, and the floor – on the process?*

6. Write a VBA code to evaluate a barbell-bullet / butterfly spread. Assume that $y_0 = .04$, $\sigma = .008$ and

$$y_t = y_{t-1} + \sigma \cdot \sqrt{\Delta t} \cdot Z_t; \quad Z_t \sim \mathcal{N}(0,1)$$

subject to:

$$y_t < .12$$

$$y_t > .005$$

Every quarter, for the next 10 years, construct a 0-net investment portfolio by buying \$500,000 of the 1-year bill and \$500,000 of the 20-year STRIPS while simultaneously selling short \$1,000,000 of the 10.5-year STRIPS. Close the position from the previous quarter at the end of each quarter and keep track of your gains and losses each quarter. Have you code report the average and minimum P&L at the end of 10 years for 10,000 simulated draws from the interest rate process.

Construct a Monte Carlo analysis of your total P&L over the 10-year period.

7. *Quiz Implications: You should be able to set up a barbell-bullet / butterfly spread trade using 3 STRIPS, and evaluate how the value of this 0-net investment portfolio changes over time – using your calculator. You should be able to explain the effect of changing σ on the trade. You should be able to explain Monte Carlo – how do we program it and how it is used, and what it can tell us.*