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P&L Attribution Analysis: Basics

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These notes are designed to accompany the spreadsheet PCARV for FIN 542.
We have built a portfolio with the following characteristics:

1. It is designed to profit by buying securities that we believe are trading rich, and shorting securities that we believe are trading cheap.
   - These beliefs about the market mispricing the securities mean that we have a model in mind.
   - We have to be explicit about our model.
2. The portfolio has 0 exposure to the sources of risk.
   - In order to achieve this objective, we also need a model to delineate:
     (a) What the sources of risk are.
     (b) How each of our securities' prices is affected by each source of risk.
   - Notice that once again we have to be explicit about our model.
3. Notice that the portfolio defined on Rows 78 - 80 in the spreadsheet meets these objective.
4. The model is a rather naïve one that says that the yield on any strip should come from the 2-factor model. As we discussed in class, the model does not have any dynamics, nor does it explain how prices could become out of line in the first place. (This latter point may be asking too much.)
5. This exercise is an example of *backtesting*.
6. We form this portfolio using information that was available on March 9, 1995. In the spirit of backtesting, we have to be careful to put ourselves in the shoes of a trader *at the time of the trade*. Obviously we know a lot more now, but we try to simulate what a trader could have done on this date.
7. Putting on the trade includes consideration of the financing arrangements for long positions and borrowing the securities that we have to short in the repo market.
8. In this example, we buy the 5-year and 25-year strips:
   (a) Call our dealer to buy the strips.
   (b) Apprise tri-party repo bank (JPMorgan-Chase) that we will borrow the $ amount of these purchases in this market.
   (c) Later in the day, transfer the securities that we bought to the tri-party repo bank to collateralize our loan.
   (d) At that point our account will be credited for the cash.

(e) We have to roll-over this position every day we hold our portfolio.

(f) The example assumes that the haircut is 0 and the repo rate is constant at 4.45% in this case.

9. We sell short the 15-year strip. To do this we call our dealer asking him to find the strip to borrow. Once he finds them, we sell and then we place the proceeds from the sale in a securities lending repo. In the repo market we are lending cash and we have possession of the strips as collateral. As with the financing repo, we assume that there is no haircut, and that the 4.4% rate remains constant over our holding period. Note in this case we do not use tri-party repo, since we need a specific strip as collateral. We also assume that the strip is not trading on special in repo. (If it were, then we would earn a lower rate on our cash loan.)

10. So now we have the 3 securities in our portfolio, and with the passage of time we hope to rake in the profits.

11. We now look at closing the position on April 20, 1995–6 weeks after we put on the position. So we need the yields on the 3 strips on that date.

12. The spreadsheet shows that we sell the strips that we own, and buy the strip that we shorted to close the positions. We also total our costs and returns in the repo markets.

13. The spreadsheet shows that we lost $1,666,160.

14. *P&L Attribution analysis* refers to our attempt to isolate the reason we have this loss.

15. This analysis looks at three specific effects:

    (a) Carry. In this case we have a net long position in 5- and 25-year strips that yield over 7%. Our financing costs are less than 5%. So we are (net) borrowing short-term and lending long-term in a setting with an upward-sloping yield curve. As such we expect that we should earn a positive return. Carry is meant to convey how we earn or lose money with the passage of time.

    (b) Factor Exposure (systematic risk). This should be fairly low since we hedged factor exposure. If there are fairly large changes in the factors then we could either make money or lose money, if the second derivatives are not 0. This is because as the factors change, the hedge ratios also change. We are not rebalancing this portfolio dynamically, so that these effects can affect our position.

    (c) Convergence. This is the impact of the idea behind the trade. If the strips that we think are undervalued increase in price–relative to the model, and the securities that we think are overvalued increase in price–relative to the model, then we will make money.

    (d) Cross Effects. Especially for positions that are open for more than a few days, this effect is likely to be fairly large. There really are cross effects. Another reason is that with the passage of time, the nature of the securities changes. For example, after 6 weeks a 5-year strip is a 4-year, 10.5-month strip. It may be priced differently from a 5-year strip–even under the model. As you look at how we compute the other 3 components, think of all the things that we have not accounted for. All of that will be included in this category.

16. Computational Issues.

    (a) Carry. Here I assume that the yields on all strips do not change over the period. In cells G11 - G115, we ask the hypothetical question: What would the prices on all of the strips be on the day we close out the position, if their yields were identical to what they were when we put on the position? Cells C150 - C153 compute the change in the prices of the 3 strips from this effect. Cells N150 - N153 contain the $

change in our positions of each strip. Note, that by definition, this will be positive for every long strip and negative for every short strip. Cell 154 sums these over the 3 strips. Financing is also part of carry, so we add the total effects of financing (in Cell E126) to compute the total carry. This is reported in Cell C156. This number is positive because we are net long, the yield curve is upward sloping, and we use overnight financing to finance long-term strips.

(b) Factor Exposure.
   i. Compute the factor realizations on the day we close out the position. This is done in Cells C136 and C137.
   ii. Then we ask what would the prices be on the strips, *under the model*–if these were the factors on the day we put on the position. This is done in Cells J140 - J142.
   iii. Cells C163 - C165 compute the difference between this model implied price and the original model implied price for all 3 strips.
   iv. Cells N163 - N165 scale this by the position in each strip.
   v. Cell C167 adds this across the 3 strips.
   vi. This does not account for the passage of time, or the possibility that we have the wrong model.

(c) Convergence.
   i. Here we establish the ratio between actual and model implied prices when we put on the position and when we take it off. As with Factor Exposure, we do not account for the passage of time.
   ii. Cells M140 - M 142 compute the ratio of the model implied price to the actual price for each strip, on the day we close the position.
   iii. Cells P140 - P142 compute the ratio of the model implied price to the actual price for each strip, on the day we established the position. (Note that we bought the strips whose ratios here are less than 1, and we shorted the strip whose ratio is greater than 1. Our "model" says that these will all be 1.)
   iv. Cells M174 - M176 multiplies the ratio of the model to the actual price on the day we close the position times the original price, minus the original price, for each strip.
   v. Cells N174 - N176 scale this by the numbers of each of the strips.
   vi. Cell C178 adds these over the 3 strips.
   vii. Note, if the model actually held perfectly on April 20, 1995 (when we close the position), then we would have made a positive profit of $6,889,585 from convergence. (**You should be able to compute this.**)

(d) Cross Effects. This is defined as the difference between our actual profit or loss and the sum of the above 3 components of that performance.

Exercise. The spreadsheet includes profit and loss for closing out the position at different dates in Cells M121 - N 130. You should verify these values, and do attribution analysis in each case. Note that we use the Lookup command from Excel to fill in the market data from the "Data" worksheet–based on the number of weeks we hold the position (in Cell I105).

This panel also highlights the problem of optimal closing out timing. If we were 100% confident in our model, then we would keep our position open until it made money. It is obvious that the quality of the hedge will deteriorate. (At the limit, recognize that in 5 years, the 5-year strip matures so that we are left holding only 2 strips.)

On Quiz 12 for November 21, I can ask about the concepts behind attribution analysis, and the motivation behind the convergence trade. I could give you factor loadings, observed yields and factor realizations on a date when a position is put on, and on the date it is liquidated, and ask you to compute the convergence component of P&L.

I could give you repo rates, and ask you to compute the carry component.

I could give a position, factor loadings, and factors on the two dates, and ask you to compute the convergence component.