Overview and Facts

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Organizing Themes

In these 5 weeks we will explore the broad area of valuing derivatives in equity markets.
This is motivated in large part by the Black-Scholes-Merton option pricing approach.
Advantages of Continuous-time modeling:

- Theory: Access to powerful tools, esp. Itô's lemma.
- Empirics: Dynamic Consistency.

Brownian Motion 1.

Origins:

- Botanist Robert Brown – 1828.
- Albert Einstein – 1905.
- Louis Bachelier – 1900.
- Formality: Norbert Weiner – 1931.

Properties:

- Normal Increments: $B(t) - B(s) \sim \mathcal{N}(0, t - s)$

Equity Returns

So the first question that we ask of the data is whether returns are (log-)normally distributed.

- Daily returns are leptokurtic. – Fatter tails than we would see under the normal distribution.
- Leptokurtosis diminishes in the differencing interval. (Reject stability under addition.)
- While tail thickness is the same for individual stocks and indices, skewness is not:
  - Individual stock returns are positively skewed.
  - Index returns are negatively skewed.

Return Dynamics

The second question concerns the dynamic behavior of stock returns.

- Return variance is higher when the market is open.
- Volatility is clustered.
- Trading volume dynamics resembles trading volume.
- Material bid-ask bounce means negative serial correlation in returns evident even at monthly frequency.
  - Implication:: Roll (1983); Blume and Stambaugh (1983) :: To form e.g., weekly portfolio returns compound individual stocks before aggregating.

Local Time

Volatility clustering can explain the leptokurtotic pattern in returns. Peter Clark (1971) suggests that daily returns should be modeled as a mixture of normals. This is modeled more formally in Tauchen and Pitts (1983), and linked to volatility clustering by Lamoureux and Lastrapes (1991).
Intuition: What would happen if Brown's son randomly (and surreptiously) changed the temperature under his colloidal fluid every few days?
Latest incarnation: VPIN (Easley, López de Prado, and O'Hara 2012).
Problem: Volume is not analogous to the temperature under Brown's in the sense that the temperature is exogenous. Presumably the direct analog in financial markets is something like information flow.

Step Back :: Methodology

Let's pause to consider what we did. We started with a basic theory of returns due to Bachelier. This has implications for:

- (Unconditional) Return Distribution.
- Return Dynamics (Markov property).

We document that returns do not conform to these restrictions, and think of what modifications are needed to the theory to give rise to that behavior.