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Corporate Equity

Corporate capital consists of debt and equity. Debt entails a contractual obligation to repay interest and principal. By contrast, equity is an ownership stake–the company does not owe equity holders anything. Equity has a "residual claim–" the right to everything the company has once all contractual obligations are paid. Equity is commonly called stock.

Companies "go public" when they issue (public) equity for the first time. This is called an initial public offering (IPO). Similar to debt, companies access public equity markets via an intermediary (investment bank), which underwrites the offering. Traditionally equity involves control. A share of stock entitles the owner to a vote. Many tech companies (famously Facebook and Google) have multiple classes of stock so that the management team (entrepreneurs) does not have to give up voting control by accessing public equity.

We define the market capitalization of equity as the number of shares outstanding times the price per share. Major stock market indices are value-weighted (e.g., S&P 500, Russell 1000, CRSP VWI). The weight of a company in the index is equal to its market capitalization divided by the sum of the market capitalizations of all of the index components. We define the (arithmetic) return on a stock in period t:

$$r_t^a = \frac{p_t + d_t - p_{t-1}}{p_{t-1}}$$

Alternatively, we prefer to use log returns, just as we preferred to work with continuously compounded returns in FIN 510A.

$$r_t = \ln(p_t + d_t) - \ln(p_{t-1}) = \ln(1 + r_t^a)$$

Log returns are consistent with keeping track of positions over time. Consider the following example: On January 31, 2023 KRO has a market price of $65 per share. On Feb 28, 2023, KRO's market price is \$130 per share, and on March 31, 2023 the price is \$65 per share. KRO paid no cash or stock distributions over this 2-month period. Obviously, if we bought 100 shares of KRO on January 31, 2023 and held the position through March, our 2-month return is 0, as is our average monthly return. But consider the 2 monthly arithmetic returns: In February, our arithmetic return is: $r^a = \frac{130-65}{65} = 1$. We made 100% in the month. In March we made: $r^a = \frac{65-130}{130} = -0.5$. We lost 50%. Using these arithmetic returns we would incorrectly conclude that we had an average rate of return of 25% over the 2-month holding period.

Consider the log returns: In February, our return is: $r = \ln(130) - \ln(65) = \ln(1+1) = .6931$. We made 69.31% in the month. In March we made $\ln(65) - \ln(130) = \ln(1-0.5) = -.6931$. We lost 69.31% in the month. The average log return is 0, and the sum of the 2 monthly log returns is 0.

d_t is the cash dividend paid between dates $t-1$ and t. A dividend leaves the stock on the ex-dividend date. Traditionally in the US (established) companies pay cash dividends on a quarterly basis. Recently stock buy-backs are a more popular way of distributing cash to shareholders than dividends. One reason for this is that traditionally it is taboo for companies to cut their cash dividends.

We also have to take stock distributions into consideration. For example, a company may undertake a 2-for-1 stock split. If I have 500 shares prior to this 2-for-1 split, then I will have 1,000 shares when the split takes effect. Consider a 10% stock dividend. If I have 300 shares before this stock dividend, then I have 330 shares when the stock dividend takes effect. Let s_t be the percent of shares increased by the distribution–for example $s_t = 1$ in the case of the 2-for-1 stock split and $s_t = .1$ in the case of the 10% stock dividend. In a period where there is a stock distribution, the (log) return is computed using a distribution adjustment for the pre-distribution price: $p^a_{t-1} = \frac{p_{t-1}}{1+s}$. Then we construct the stock log return in period t:

$$r_t = \ln(p_t + d_t) - \ln p^a_{t-1}$$

For example, Apple Computer made a 7-for-1 stock splt on June 9, 2014, and it paid no cash dividends in that month. Apple's stock price on (Friday) May 30, 2014 was \$633.00 and on June 30, 2014 \$92.93. In this case $s = 6.0$ and Apple's stock return in June 2014 is: $r = \ln(92.93) - \ln(\frac{633}{7}) = 2.729\%$. The corresponding arithmetic return: $r^a = \exp(r) - 1 = 2.766\%$.

Once we have a record of stock returns (for example 10 years of monthly log returns), we process them statistically in the following ways.

Mean / Expected Return $E(r) = \frac{\sum_{t=1}^{T} r_t}{T}$ This is the average log return per period (e.g., months).

Return Variance $Var(r) = \frac{\sum_{t=1}^{T} (r_t - E(r))^2}{T}$ This is the expected per period squared deviation from the mean log return.

Return Standard Deviation $\sigma(r) = \sqrt{Var(r)}$ This places the variance in the same unit as returns themselves so it is a commonly used measure of "scale" or dispersion.

Much more important than own measures of risk are cross-measures of risk such as the covariance between stock i and stock j and the beta of a stock.

Covariance between stock i and stock j $Cov(r_i\ ,\ r_j) = \frac{\sum_{t=1}^{T} (r_{i,t} - E(r_i))(r_{j,t} - E(r_j))}{T}$

Correlation between stock i and stock j $\rho(r_i\ ,\ r_j) = \frac{Cov(r_i\ ,\ r_j)}{\sigma(r_i) \cdot \sigma(r_j)}$

This is the standardized co-movement between two random variables. It must lie between -1 and 1. If $|\rho| = 1$, then the two variables are perfectly correlated (that is there is only 1 source of randomness). If $\rho = 0$, then the variables are independent.[1]

Stock's beta $\beta_i = \frac{Cov(r_i\ ,\ r_m)}{Var(r_m)}$

When we form N stocks into a portfolio the portfolio mean log return is:

$$E(r_p) = \sum_{i=1}^{N} w_i \cdot E(r_i)$$

w_i is the weight in stock i – that is the percentage of the portfolio's market capitalization due to stock i.

The variance of the portfolio's log returns is:

[1]Technically measuring scale with squared deviations does depend on distributional properties of the random variables. We will generally assume that stock returns are normally distributed, which means that we can describe the entire distribution using the mean and variance. And if we have several stocks, then we can describe the entire multivariate distribution using the mean vector and variance-covariance matrix.

$$Var(r_p) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i \cdot w_j \cdot cov(r_i\ ,\ r_j)$$

Notationally, $cov(r_i\ ,\ r_j)$ is the $Var(r_i)$, when $i = j$. (This is a quadratic form.) This is an extremely important result. The portfolio mean is the average of the mean returns of its components. However the variance will be less than the average variance as long as the stocks in the portfolio are less than perfectly correlated.

We generally think of the stock market as the value of wealth in the economy, so the systematic risk of a stock is measured by its beta. That is the only portion of the stock's return variance that compensates its owner for risk.

The risk premium on the market is: $E(r_m) - r_f$. So we can decompose the cost of equity as: $E(r_i) = r_f + \beta_i \cdot (E(r_m) - r_f)$.

Notice that this is entirely consistent with the intuition that we developed for cat bonds, as cat bonds have a beta of zero (their payoff is not related to the state of the economy), and therefore their required rate of return is just r_f.

This is also fully consistent with the intuition we developed for corporate debt. We established that the expected return on corporate debt equals the risk-free rate plus a risk premium. Because we have a (forward looking) yield for debt, we decompose the credit spread into a hazard rate (which simply compensates for expected loss) plus a risk premium. We use a rule of thumb based on historical data that $\frac{1}{2}$ of the credit spread is the hazard rate and the other half is the risk premium. The risk premium reflects the extent to which bankruptcy is correlated with the state of the economy.

The variance-covariance matrix is a 2-dimensional array that contains the variances of the stocks along the diagonal and the covariance between stock i and j in row i and column j. For example, suppose we have three stocks whose variances are: $Var(r_1) = 0.04$, $Var(r_2) = 0.2025$, and $Var(r_3) = 0.1089$. And the pairwise correlations are: $\rho(r_1\ ,\ r_2) = 0.45$, $\rho(r_1\,, r_3) = 0.22$, and $\rho(r_2\,, r_3) = 0.30$, then the variance-covariance matrix is:

$$V = \begin{bmatrix} 0.04 & 0.0405 & 0.01452 \\ 0.0405 & 0.2025 & 0.04455 \\ 0.01452 & 0.04455 & 0.1089 \end{bmatrix}$$

Notice that the variance-covariance matrix is symmetric – that is, the element in row i, column j is the same as in row j, column i. This is because $\rho(r_1\ ,\ r_2) = \rho(r_2\ ,\ r_1)$.

For those familiar with matrix algebra, note that we can define a vector of portfolio weights:

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

The weights must sum to 1. A negative weight means a short position.

And then we can write the variance of a portfolio: $w'Vw$, which is a quadratic form.

2 stock example

Let's apply the expression for the variance of a portfolio to the case of 2 stocks. Consider a portfolio with 65% invested in ITT, that has an expected monthly return of 1% and a monthly return standard deviation of 10%; and 35% invested in LHX, that has an expected monthly return of 0.6% and a monthly return standard deviation of 8%. The correlation between LHX and ITT's returns is -0.1. We remember that the covariance between two random variables is the product of the correlation between the two and the two standard deviations.

We compute the expected monthly return on the portfolio: $E(r_p) = 0.65 \cdot .01 + 0.35 \cdot 0.006 = 0.0086$. The portfolio's monthly expected return is 86 basis points. The portfolio variance is: $0.65^2 \cdot 0.10^2 + 0.65 \cdot 0.35 \cdot -0.1 \cdot 0.1 \cdot 0.08 + 0.35 \cdot 0.65 \cdot -0.1 \cdot 0.1 \cdot 0.08 + 0.35^2 \cdot 0.08^2$. Note that the second and third terms in the quadratic expression are the same, so we can rewrite this: $\text{Var}(r_p) = 0.65^2 \cdot 0.10^2 + 2 \cdot 0.65 \cdot 0.35 \cdot -0.1 \cdot 0.1 \cdot 0.08 + 0.35^2 \cdot 0.08^2 = 0.004645$. Which means the portfolio standard deviation is: 6.8154% per month.

Note that the portfolio's standard deviation will be simply the weighted average of the two standard deviations only in the special case of $\rho = 1$. In this case there are no benefits from diversification. However, when the correlation is less than 1, then the portfolio standard deviation is less than the weighted average of the two standard deviations. In the ITT / LHX example, note that the portfolio standard deviation is less than the standard deviation of both of the stocks.