Introduction to Vectors and Matrices for Finance
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Numbers that we are used to working with are called scalars. A vector is a set of numbers ordered in a column. Example of vectors that we encounter include stocks expected returns and portfolio weights. For example consider a portfolio of 3 stocks with weights: $w_1 = .3$, $w_2 = .4$, $w_3 = .3$. This means that 30% of the portfolio is invested in Stock 1, 40% in Stock 2, and 30% in Stock 3. In this case, w is a vector of length 3, and we can write: $w = \begin{bmatrix} .3 \\ .4 \\ .3 \end{bmatrix}$

Similarly, suppose that the expected monthly returns on Stocks 1, 2 and 3 are: .01, .0085, and .007, respectively. Then we can write: $E = \begin{bmatrix} .01 \\ .0085 \\ .007 \end{bmatrix}$

Here the vector E is the 3-vector of expected returns. We know that the expected return on the portfolio is the weighted average of the components stocks' expected returns: $E(r_p) = w_1 \cdot E(r_1) + w_2 \cdot E(r_2) + w_3 \cdot E(r_3)$.

We can also express this concisely using vector multiplication: $E(r_p) = w'E$. w' is the transpose of the column vector w, which is thereby a row vector: $w' = \begin{bmatrix} .3 & .4 & .3 \end{bmatrix}$

Multiplying vectors requires conformance. We multiply a 1 × 3 row vector (which has 1 rows and 3 columns) by a 3 × 1 column vector (which has 3 rows and 1 column), and the result is 1 × 1 – i.e., a scalar. We have: $w'E = .0085$. The resulting scalar is the sum of the inner product of the vectors.

A vector is also called a 1-dimensional array (especially in coding). A matrix is a 2-dimensional array, and works in the same way as a vector. In portfolio contexts, we encounter the variance covariance matrix. In our 3 stock portfolio, for example, the variance covariance matrix is $V = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & \sigma_{1,3} \\ \sigma_{2,1} & \sigma_{2^2} & \sigma_{2,3} \\ \sigma_{3,1} & \sigma_{3,2} & \sigma_3^2 \end{bmatrix}$

Our notation is that σ_i^2 is the variance of the returns on stock i, and $\sigma_{i,j}$ is the covariance between stocks i and j. And of course, $\sigma_{i,j} = \sigma_{j,i}$. So this is a symmetric matrix. It is a 3 × 3 square matrix.

There are $\frac{N \cdot (N+1)}{2}$ unique elements in a symmetric $N \times N$. In this case, 3 variances and 3 unique pairwise covariances. We know that the portfolio variance can be written:

$$V_p = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i \cdot w_j \cdot \sigma_{i,j}$$

As N gets large, this becomes tedious. So we can use vector and matrix multiplication to simplfy things. Especially since we can use these operators directly in Excel.

We can also express the portfolio variance as $V_p = w'Vw$.

As we know, w' is a 1 × 3 row vector and V is a 3 × 3 square matrix. So first we compute $w'V$, which is a 1 × 3 row vector. Then we post-multiply this by w, a 3 × 1 column vector, which yields a scalar. I say post-multiply, since with matrices and vectors it matters which is first (we could not compute wV, for example – this lacks conformance).

We can convince ouselves in Excel that we get the same using either approach.