Introduction to Finance - II

Quiz on multivariate stock returns Key

1. We have 120 returns on each of 2 stocks in columns in Excel. We want to compute the covariance between the returns.

   (a) We need the mean returns on both stocks. Use =Average(C3:C122) (in C125, e.g.), and =Average(D3:D122) (in D125, e.g.)

   (b) We need each of the 120 deviations from the mean. In column F: =C3-C125 (rows 3 through 122) and in Column G: =D3-D125 (rows 3 through 122).

   (c) Next we will get the squared deviations in each month on each of the 2 stocks. Let's put those for LRCX in Column I and for MXIM in Column J. So I3 is =F3ˆ 2 and J3 has =G3ˆ 2 $\cdots$ through row 120.

   (d) The average of the 120 squared deviations is the variance. So =Average(I3:I122) will be the variance of LRCX and =Average(J3:J122) will be the variance of MXIM. We also compute the standard deviations of the monthly returns by taking the square roots of the variances: =SQRT(I125) will be LRCX's monthly return standard deviation. And =SQRT(J125) will have MXIM's standard deviation.

   (e) The covariance between the monthly returns on LRCX and MXIM is the average of the cross-products of the deviations. To get this, we can construct a column that contains these cross-products. For example, Column L: Row 3 would have: =F3*G3 through row 122: =F122*G122.

   (f) The covariance is then =average(L3:L122). Lets place this in Cell L125.

   (g) And the correlation (which has to lie between -1 and +1) is the covariance divided by the product of the two stocks' standard deviations. For example in Cell L126: =L125/(I125*J125).

2. This question can be answered with a calculator. LRCX has an expected monthly return of 0.45% and a monthly return variance of 0.016. MXIM has an expected monthly return of 0.6% and a monthly return variance of 0.0088. The correlation between the two stocks' returns is 0.43. Form a portfolio with 60% in LRCX and 40% in MXIM.

   (a) $w_1 = .6$ and $w_2 = .4$. Portfolio mean monthly return: $.6 \cdot 0.0045 + .4 \cdot 0.006 = .0051$. Or 0.51% per month. Portfolio monthly return variance: $.6^2 \cdot .016 + .4^2 \cdot .0088 + 2 \cdot .6 \cdot .4 \cdot (.43)(\sqrt{.016})(\sqrt{.0088}) = 0.009617$. (Since the covariance is the correlation times the product of the two standard deviations.) So the portfolio monthly standard deviation is 9.8067%.

   (b) To address the probabilistic ranges under the normal distribution, we first annualize the mean and standard deviation. Annual mean: $12 \cdot .0051 = 6.12\%$. And the annualized return standard deviation: $\sqrt{12} \cdot .098067 = 33.9714\%$. The ranges then are:

Range	Low Return	Expected Return	High Return
68%ile Range	$.0612 - .339714$	.0612	$.0612 + .339714$
95%ile Range	$.0612 - 2 \cdot .339714$	.0612	$.0612 + 2 \cdot .339714$
69%ile Range	$.0612 - 3 \cdot .339714$	.0612	$.0612 + 3 \cdot .339714$

   (c) If the correlation between these two stocks is 0, then the portfolio's expected return is not affected, and the variance becomes: $.6^2 \cdot .016 + .4^2 \cdot .0088 + 2 \cdot .6 \cdot .4 \cdot (0)(\sqrt{.016})(\sqrt{.0088}) = 0.00772$. And the portfolio's monthly standard deviation is 8.4664%.

   (d) Diversification is a "free lunch." By combining stocks into a portfolio, the portfolio's expected return is a simple weighted average of the individual stocks' mean returns. However the variance is less than the analogous average of the individual stock return variances – and the extent to which it is less depends on the correlations between the component stocks. The smaller are the correlations, the bigger the gain from diversification.