Block 1 Quiz 1 – A single stock's returns.

Step 1. Collect data on prices, dividends, and stock distributions from a source such as CRSP. Here I have monthly data from January 2007 through December 2016, on 3 stocks: Applied Materials (AMAT), Lockheed Martin (LMT), and 3M (MMM). Note that in the US, most companies pay dividends on a quarterly basis. Note too that during this period none of these 3 stocks had a stock distribution (stock split or stock dividend).

Step 2. Construct the returns from that data. Notice that we need 2 adjacent periods to construct one return, so we lose one observation (i.e., the first return is from Jan 31 2007 through Feb 28, 2007).

Step 3. Obtain the expected return on each stock, which is the mean return.

Step 4. Construct the monthly return minus the mean return each month. (These should sum to 0).

Step 5. Square the monthly return minus the mean return each month. Squaring changes the difference to a distance. The mean of these monthly distances is the return variance. The square-root of the variance is the return standard deviation.

Note that the mean, variance, and standard deviation are relevant to the frequency of the data. We originally worked with monthly data, so the mean return is the expected monthly return, and the standard deviation is the monthly return standard deviation. We can annualize these by multiplying the mean and the variance by 12 (therefore the monthly standard deviation is multiplied by the square-root of 12).

Discussion:

We sometimes approximate the probabilistic distribution of returns using a normal distribution. If we know the current stock price, and the mean and standard deviation of the returns, then we can quantify possible outcomes in the future. For example, if we have 100 shares of Applied Materials on December 30, 2016, then the expected value of our position after one year is:

100*32.27*(1+12*.01063) = 3638.62

The 68% probability range is:

$$3638.62 \cdot \left(1 - \sqrt{12} * 0.0879\right) \text{ to } 3638.62 \cdot \left(1 + \sqrt{12} * .0879\right)$$

The 95% probability range is:

$$3638.62 \cdot \left(1 - 2\sqrt{12} * .0879\right) \text{ to } 3638.62 \cdot \left(1 + 2\sqrt{12} * .0879\right)$$

The 99% probability range is:

$$3638.62 \cdot \left(1 - 3\sqrt{12} * .0879\right) \text{ to } 3638.62 \cdot \left(1 + 3\sqrt{12} * .0879\right)$$

Quiz 2

Next we consider a portfolio of stocks. If we are willing to assume that a normal distribution can describe the probabilistic structure of a stock's return, then we can also assume that all stocks jointly can be described by a multivariate normal distribution. This means that in addition to knowing the mean and standard deviation of each stock, we also need the pairwise correlations between all pairs of stocks. Remember that the variance is the average distance from the mean return. The covariance is the average product of two stocks' differences from their respective mean returns each period. So consider Applied Materials (AMAT) and Lockheed Martin (LMT) in our spreadsheet. Column U contains the cross products of these deviations. The average of these is 0.00172. This is the covariance between AMAT and LMT. (Of course it is also the covariance between LMT and AMAT.) Like the variance, this measure is not in a natural metric. We can only see that the two stocks are positively correlated (since the covariance is positive).

A more natural measure of co-variation is the correlation between 2 stocks. The correlation is the covariance divided by the product of the 2 standard deviations. This standardizes the covariance and the correlation must be between -1 and +1. The spreadsheet shows that the correlation between AMAT and LMT is 0.3323. Loosely speaking this means that the 2 stocks tend to move in the same direction, and about 33% of their movements are shared.

Now we consider a portfolio that contains (only) AMAT and LMT. A portfolio is defined by the weights on each stock. The weights are based on the proportion of the portfolio's market value.

For example, consider a portfolio with 700 shares of AMAT and 100 shares of LMT. The value of the AMAT holding is 700 * 32.27 = $22,589, and the value of the LMT position is 100 * 249.94 = $24,994. So the value of this portfolio is $47,583, and the weight in AMAT is 22589/47583 = .4747, and the weight in LMT is 24994/47583 = .5253. We should verify that the weights sum to 1.

The portfolio's expected return is simply the weighted average of the expected returns of the component stocks. So our portfolio's monthly expected return is: .4747 * .0106 + .5253 * .0125 = .0116. The portfolio's return variance is: $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{1,2}$. The intuition for this formula includes the fact that the variance of a constant, w, times a random variable is $w^2 \cdot \sigma^2$.

The notation here is σ_p^2 is the portfolio return variance, w_1 is the weight in AMAT (stock 1), and w_2 is the is weight in LMT (stock 2). $\sigma_{1,2}$ is covariance between stocks 1 and 2. Plugging in the values from the spreadsheet, the variance of our portfolio is 0.003557. This means the portfolio standard deviation is .0596.

The *Diversification* worksheet shows how portfolio risk and return are related to the correlation between 2 stocks. Here we consider combining two stocks. Both have an annual expected return of 8%, and annual return standard deviation of 30%. We consider an equally-weighted portfolio, where ½ of the portfolio is invested in each of the 2 stocks. When the correlation between the 2 stocks is 1, then there are no diversification benefits. In this case, the 2 stocks are actually the same. As the correlation shrinks the benefits from diversification become larger. Note too that the portfolio expected return does not depend on the correlation.

An overarching theme in finance is that if risk can be diversified, then the market will not compensate for exposure to that risk. This is why the expected returns on catastrophe bonds do not include a risk premium. (Recall that the expected return on a weather-related cat bond is the risk-free rate.)

Quiz 3

The Capital Asset Pricing Model is built on several assumptions including the arbitrage premise that risk that can be diversified away will not be priced, and that the stock market is a proxy for the state of the economy. The variance of a stock (i) can be split into two parts: diversifiable and non-diversifiable. $\sigma_i^2 = \beta_i{}^2 \cdot \sigma_m^2 + \sigma_{i,\epsilon}^2$

This split is accomplished by a linear regression of the stock's returns on the market returns. The regression coefficient, beta, is the covariance between the market and the stock returns divided by the variance of the market return. The accompanying spreadsheet uses the CRSP value-weighted index as the market portfolio, and computes beta for each of the 3 stocks: AMAT, 1.2711; LMT, 0.8089; and 3M, 0.8477.

Looking back on the portfolio variance, we know that the regression decomposition looks like this: $\sigma_i^2 = \beta_i{}^2 \cdot \sigma_m^2 + \sigma_{i,\epsilon}^2 + 2\mathrm{cov}(r_m, \epsilon)$ The key to the linear regression is independence between the residual and the independent variable, so that $\mathrm{cov}(r_m, \epsilon) = 0$.

Following on how we ascertained the expected return on bonds, the CAPM says that the expected return on a stock equals the risk-free rate plus the risk premium. The market risk premium is the expected return on the market minus the risk-free rate. The stock's risk premium then is: $\beta_i \cdot \{E(r_m) - r_f\}$. Therefore the expected return on stock i is:

$$E(r_i) = r_f + \beta_i \cdot \{E(r_m) - r_f\} .$$

This may seem contradictory, since we earlier defined the expected return on a stock as simply the average of its returns in the past. That definition is purely statistical. The link to beta is theoretically motivated, and more easily estimated. Furthermore, it distinguishes the specific market pattern in the sample from expected returns.

So if the risk-free rate is 3.5% (we can use the yield to maturity on the 30-year US Treasury Bond), and the expected return on the market is 8.5%, then the expected returns on the 3 stocks from the spreadsheet are:

AMAT: $3.5\% + 1.2711 \times 5\% = 9.8557\%$

LMT: $3.5\% + 0.8089 \times 5\% = 7.5443\%$

MMM: $3.5\% + 0.847\ 5\% = 7.7387\%$

Since these are the expected returns on the stock, they are also the costs of equity capital to the issuers.

A company's weighted average cost of capital averages the after-tax costs of debt and equity according to the proportions of each in the capital structure.

3M has 13 billion in debt, a AA credit rating, and a credit spread of 50 basis points. This means that its after-tax cost of debt is (.79) (.035 + .0025) = 2.9625%. The market cap of its equity is $104 billion. So its weighted average cost of capital is (13/117) (.029625) + (104/117) (.077387) = 7.2080%.